Random Response of Inelastic Space Structures Subjected to Bi-Directional Ground Motion

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By Teizo FUJIWARA and Takashi HOSOKAWA

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Abstract

The objective of this paper is to clarify the aseismic safety of the ultimate state of space structures. A method of stochastic earthquake response analysis is presented, which extends the formulation of Y. K. Wen et al. by introducing a yielding condition and its associated flow rule into the differential equations of hysteresis rule.

In order to study the effect of bi-directional ground motion on the responses such as ductility, hysteretic energy dissipation of space structures, the parametric analysis of various frequency ratios and strength ratios between perpendicular axes of structures is performed. The responses under bi-directional ground motion are compared with those under uni-directional motion and the effect of deterioration of hysteresis are also studied.

As the results of the analysis, the following interesting conclusions were obtained: 1) Standard deviation of ductility response is affected by the frequency characteristics of space structure, especially in the smaller frequency direction, along which smaller ductility response is likely under uni-directional ground motion, 2) The effect of deterioration of hysteresis is remarkable in both axes of structure, especially in the case of bi-directional ground motion, 3) The effect of bi-directional ground motion is greater on the ductility response in the stronger axis of structure than in the weak axis, and when considering the deterioration of hysteresis, displacement responses in both directions increase but the total hysteretic energy dissipation under bi-directional motions is equal or less than that under uni-directional motion.

1. Introduction

The ultimate aseismic safety of space structures subjected to multi-directional ground motions must be studied under the multi-axial force state. Since N. C. Nigam introduced the interactive effect of bi-axial shear forces on the response analysis[1], some investigations on the analysis of space structures have been performed by various investigators using different approaches[2-7]. Recently, the author and others performed the shaking table test for the purpose of studying the interactive effects of bi-directional horizontal ground motion on the inelastic dynamic hysteresis of a single-story space frame structure and compared the results of analytical response with the test results[8,9]. The following concluding remarks were obtained from the above research: 1) Hysteresis loop obtained from the test under bi-directional input motion is considerably different from that for uni-directional input motion. 2) Test results can be followed by the numerical analysis considering the interaction effect under multi-axial forces. 3) Ductility ratio responses under bi-directional motions increase more than the response under uni-directional motion and mainly in the direction of the stronger axis of the space structure. However, from the restricted results of the deterministic analysis, general conclusions could not be obtained.

The objective of this paper is to obtain a more general tendency of the interactive effects from nondeterministic analysis. Y. J. Park, Y. K. Wen and A. H. Ang presented a method of random response analyses of hysteretic systems under bi-directional ground motion by introducing coupled differential equations of hysteretic constitutive law[10]. The method presented here is an extension of the above method, where the equations of motion considering different natural frequencies between perpendicular axes of space structures are considered and a general hysteretic formulation is introduced by considering a yielding condition and the associated plastic flow rule.

2. Method of Analysis

2.1. Equations of motion

Equations of motion of a single story space structure subjected to horizontal ground acceleration F_j are given by eq. (1).

$$M_j\ddot{U}_j+C_j\dot{U}_j+Q_j=-M_jF_j \qquad j=x,y \tag{1}$$

where M_j, C_j and Q_j are mass, damping coefficient and restoring force, respectively. U_j, $\dot{U}_j$ and $\ddot{U}_j$ show displacement, velocity and acceleration. Suffix j indicates the X- or Y-axis of perpendicular directions of a space structure. The restoring force is divided into two components: Linear component and an hysteretic component, the latter of which interacted each other according to the yield condition given by eq. (2) and **Fig.** 1.

$$Q_j=Q_j^e+Z_j \tag{2}$$

Nondimensional equations of motion can be derived by initial stiffness $\bar{K}_j$, yield strength $\bar{Q}_j=\bar{K}_j\bar{\Delta}_j$ and fundamental natural circular frequency in the X-axis ω_x of a space structure as follows:

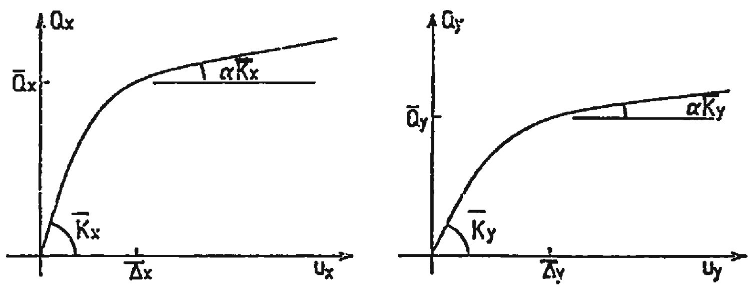


Fig. 1 Orthotropic property of system.

$$\{\ddot{\mu}\}+2h[\omega]\{\dot{\mu}\}+[\omega]^2\{\mu\}+(1-a)[\omega]\{z\}=-[\Delta]\{f\} \tag{3}$$

where
$$\{\mu\}=\{\mu_x,\ \mu_y\}^T,\ \{f\}=\{f_x, f_y\}^T,\ \{z\}=\{z_x,\ z_y\}^T$$
$$[\omega]=\begin{bmatrix} 1 & 0 \\ 0 & \omega_y/\omega_x \end{bmatrix},\ [\Delta]=\begin{bmatrix} 1 & 0 \\ 0 & \bar{\Delta}_x/\bar{\Delta}_y \end{bmatrix},\ h=C_j/2M_j\bar{K}_j$$
$$\mu_j=U_j/\bar{\Delta}_j,\ z_j=Z_j/\bar{Q}_j,\ \omega_j^2=\bar{K}_j/\bar{M},\ M_x=M_y=\bar{M}$$
$$t^2/T^2=\bar{K}_x/\bar{M},\ f_x(t)=F_x(T)\bar{M}/\omega_x^2\bar{\Delta}_x,\ f_y(t)=F_y(T)\bar{M}/\omega_x^2\bar{\Delta}_x$$

In the above equations, frequency ratio ω_y/ω_x and strength ratio $\bar{Q}_y/\bar{Q}_x=(\omega_y/\omega_x)^2\bar{\Delta}_y/\bar{\Delta}_x$ are introduced as significant parameters of the space structure.

2.2. Hysteretic characteristics of complex model

The first author of the present study presented the Ramberg-Osgood hysteresis model considering bi-axial bending moment and axial force interaction in an incremental formulation in the previous paper[4,8 9]. With this model it was possible to roughly estimate the actual hysteresis of steel.

In this paper, a deteriorating hysteresis model such as reinforced concrete structures is proposed. An constitutive equation for one-dimensional force-deformation hysteresis was presented by Y. K. Wen et al. as follows[11]:

$$\dot{Z}=[A-\{\beta\,\mathrm{sgn}(\dot{U}Z)+\gamma\}|Z|^n]\dot{U} \tag{4}$$

in which $\dot{Z}$ and $\dot{U}$ respectively represent the rates of restoring force and displacement. The parameters β, γ and n control the shape of hysteresis as shown in **Fig. 2**. Parameter A designates the initial stiffness from which stiffness varies with

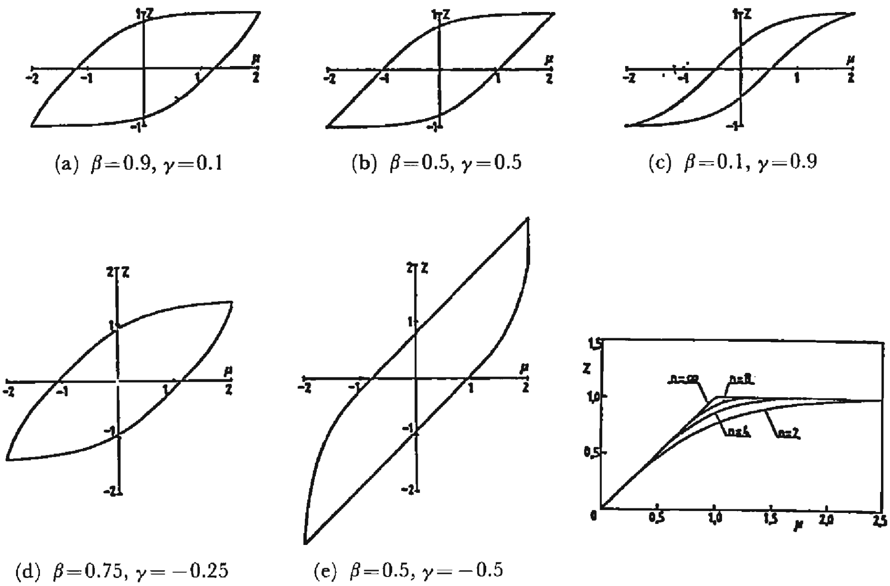


Fig. 2 Typical hystereses governed in eq. (4).

hysteretic dissipated energy E_t.

Here, nondimensional equivalent force is defined by using the following simplified yield curve.

$$z=\frac{Z}{\bar{Q}}=z\left(\frac{Z_x}{\bar{q}_x},\frac{Z_y}{\bar{q}_y}\right)=\operatorname{sgn} z(z_x^2+z_y^2)^{1/2},\ \bar{q}_j=\bar{Q}_j/\bar{Q} \tag{5}$$

The rate of plastic deformation in the j-axis is expressed according to the orthogonality condition of Reuss.

$$\dot{\mu}_j^p=\frac{\dot{U}_j^p}{\Delta_j}=\lambda\frac{\partial z}{\partial z_j}\cdot\frac{\bar{K}}{\bar{Q}}\cdot\frac{1}{\bar{q}_j^2/k_j},\ k_j=\frac{\bar{K}_j}{\bar{K}},\qquad j=x,y \tag{6}$$

Differentiating eq. (5) and assuming total deformation equal to the sum of elastic and plastic deformations, the following equations are obtained.

$$\dot{z}=\frac{\dot{Z}}{\bar{Q}}=\frac{dz}{dt}=\left\langle\frac{\partial z}{\partial z_j}\cdot\dot{z}_j\right\rangle \tag{7}$$

$$\mu_j=\mu_j^e+\mu_j^p \tag{8}$$

$$\dot{z}_j=\frac{\dot{Z}_j}{\bar{Q}_j}=(\dot{\mu}_j-\frac{\bar{K}}{\bar{Q}}\frac{1}{\bar{q}_j^2/k_j}\lambda\frac{\partial z}{\partial z_j})a_t \tag{9}$$

in which a_t is a ratio of temporal stiffness to reference stiffness and $\langle\ \ \rangle$ means the scalar product. Substituting the rate of plastic work,

$$\dot{w}^p=\frac{\dot{W}^p}{\bar{Q}^2/\bar{K}}=\left\langle\frac{\bar{q}_j^2}{k_j}z_j\dot{\mu}_j^p\right\rangle=z\dot{\mu}^p \tag{10}$$

in eq. (7), the rate of equivalent force is obtained as follows:

$$\begin{aligned}\dot{z}=\dot{w}^p\frac{dz}{dw^p}&=\frac{1}{z}\frac{dz}{d\mu^p}\left\langle\frac{\bar{q}_j^2}{k_j}z_j\dot{\mu}_j^p\right\rangle=\lambda\frac{1}{z}\frac{dz}{d\mu^p}\frac{\bar{K}}{\bar{Q}}\left\langle z_j\frac{\partial z}{\partial z_j}\right\rangle\\&=\left\langle\frac{\partial z}{\partial z_j}\dot{z}_j\right\rangle=\left\{\left\langle\frac{\partial z}{\partial z_j}\dot{\mu}_j\right\rangle-\frac{\bar{K}}{\bar{Q}}\lambda\left\langle\frac{1}{\bar{q}_j^2/k_j}\frac{\partial z}{\partial z_j}\frac{\partial z}{\partial z_j}\right\rangle\right\}a_t\end{aligned} \tag{11}$$

where μ^p shows the equivalent plastic deformation corresponding to the equivalent force z.

From eq. (11), the constant λ is obtained as

$$\lambda=\frac{a_t\left\langle\frac{\partial z}{\partial z_j}\dot{\mu}_j\right\rangle}{\left\{\frac{1}{z}\frac{dz}{d\mu^p}\left\langle z_j\frac{\partial z}{\partial z_j}\right\rangle+a_t\left\langle\frac{k_j}{\bar{q}_j^2}\frac{\partial z}{\partial z_j}\frac{\partial z}{\partial z_j}\right\rangle\right\}\frac{\bar{K}}{\bar{Q}}} \tag{12}$$

Substitution of eq. (12) into eq. (9) yields

$$\dot{z}_i=\frac{\dot{Z}_i}{\bar{Q}\bar{q}_i}=a_t\delta_{ij}\dot{\mu}_j-\frac{a_t^2\frac{k_i}{\bar{q}_i^2}\left\langle\frac{\partial z}{\partial z_j}\dot{\mu}_j\right\rangle}{\frac{1}{z}\frac{dz}{d\mu^p}\left\langle z_j\frac{\partial z}{\partial z_j}\right\rangle+a_t\left\langle\frac{k_j}{\bar{q}_j^2}\frac{\partial z}{\partial z_j}\frac{\partial z}{\partial z_j}\right\rangle}\frac{\partial z}{\partial z_i} \tag{13}$$

From eq. (4), the hardening term of the above equation is given by

$$\frac{1}{z}\frac{dz}{d\mu^p}=\frac{A}{\bar{K}}\frac{1}{z}\frac{[A-\{\beta\,\mathrm{sgn}(\dot{\mu}z)+\gamma\}|z|^n\bar{Q}^n]}{\{\beta\,\mathrm{sgn}(\dot{\mu}z)+\gamma\}|z|^n\bar{Q}^n} \tag{14}$$

By substituting eq. (14) into eq. (13) the following differential equations are obtained:

$$\begin{aligned}\dot{z}_x&=a_t\left(1-\Lambda\frac{k_x}{\bar{q}_x^2}z_x^2\right)\dot{\mu}_x-a_t\Lambda\frac{k_x}{\bar{q}_x^2}z_xz_y\dot{\mu}_y\\ \dot{z}_y&=-a_t\Lambda\frac{k_y}{\bar{q}_y^2}z_xz_y\dot{\mu}_x+a_t\left(1-\Lambda\frac{k_y}{\bar{q}_y^2}z_y^2\right)\dot{\mu}_y\end{aligned} \tag{15}$$

where

$$\Lambda=\frac{1}{\dfrac{k_x}{\bar{q}_x^2}z_x^2+\dfrac{k_y}{\bar{q}_y^2}z_y^2+\dfrac{[A-\{\beta\,\mathrm{sgn}(\dot{\mu}z)+\gamma\}|z|^n\bar{Q}^n]}{\{\beta\,\mathrm{sgn}(\dot{\mu}z)+\gamma\}|z|^{n-2}\bar{Q}^n}\dfrac{a}{a_t}}$$

a_t is assumed to be same as $a=A/\bar{K}$ and reference values are defined as

$$\bar{K}=A_0,\qquad \bar{Q}=\sqrt[n]{A_0/(\beta+\gamma)}$$

In particular, for isotropic systems, that is, $\bar{q}_i^2/k_i=1$, the above equations reduce to those apprecable to isotropic systems.

$$\begin{aligned}\dot{z}_x&=(a_t-\Lambda' z_x^2)\dot{\mu}_x-\Lambda' z_xz_y\dot{\mu}_y\\ \dot{z}_y&=-\Lambda' z_xz_y\dot{\mu}_x+(a_t-\Lambda' z_y^2)\,\dot{\mu}_y\\ \Lambda'&=[\{\beta\,\mathrm{sgn}(\dot{\mu}z)+\gamma\}|z|^{n-2}]\bar{Q}^n/\bar{K}\end{aligned} \tag{16}$$

In order to lead the equation presented cy Y. J. Park et al, it is necessary to assume $\mathrm{sgn}(\dot{\mu}z)=\mathrm{sgn}(\dot{\mu}_yz_y)=\mathrm{sgn}(\dot{\mu}_xz_x)$, announced by R. Minai[15].

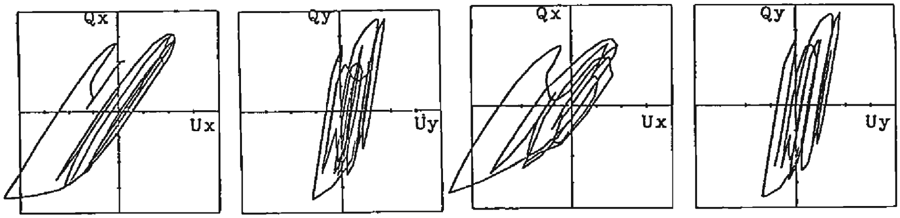
Assuming that the parameters A, β and γ are the linear functions of hysteretic energy dissipation[11],

$$\dot{e}_t=\frac{\dot{E}_t}{\bar{Q}_x\bar{\Delta}_x+\bar{Q}_y\bar{\Delta}_y}=\frac{(1-a)\left\{z_x\dot{\mu}_x^p+\left(\dfrac{\omega_y}{\omega_x}\right)^2\left(\dfrac{\bar{\Delta}_y}{\bar{\Delta}_x}\right)^2z_y\dot{\mu}_y^p\right\}}{1+\left(\dfrac{\omega_y}{\omega_x}\right)^2\left(\dfrac{\bar{\Delta}_y}{\bar{\Delta}_x}\right)^2} \tag{17}$$

the following expressions can be obtained, where A_0, β_0 and γ_0 represent initial values[10].

$$\begin{aligned}A&=A_0-\delta_Ae_t\\ \beta&=\beta_0+\delta_\beta e_t\\ \gamma&=\gamma_0+\delta_\gamma e_t\end{aligned} \tag{18}$$

Figure 3 shows two types of typical hysteretic responses for both axes; hysteresis (a) governed by eq. (15) and hysteresis (b) governed by eg. (16). The parameters of the model structure considered here are $\omega_y/\omega_x=0.5$, $h=0.02$, $A=1$, $\beta=\gamma=0.5$, $n=2$ and the acceleration wave form used here is a pair of white noise. Maximum average responses of displacements and hysteretic dissipated energy for nine pairs of bi-directional ground motion produced from white noise are shown in **Fig. 4**, where the



(a) Hysteresis based on eq. (15) (b) Hysteresis based on eq. (16)

Fig. 3 Comparison of hystereses governed by eq. (15) with that governed by eq. (16).

natural frequency ratio for both axes are chosen in the range of 0.5 to 1.5 and the strength for both axes are the same value.

As shown in those figures, the hysteresis governed from eq. (15) is a little different from that based on eq. (16). However, their maximum average responses are quite similar to each other for various kinds of structures.

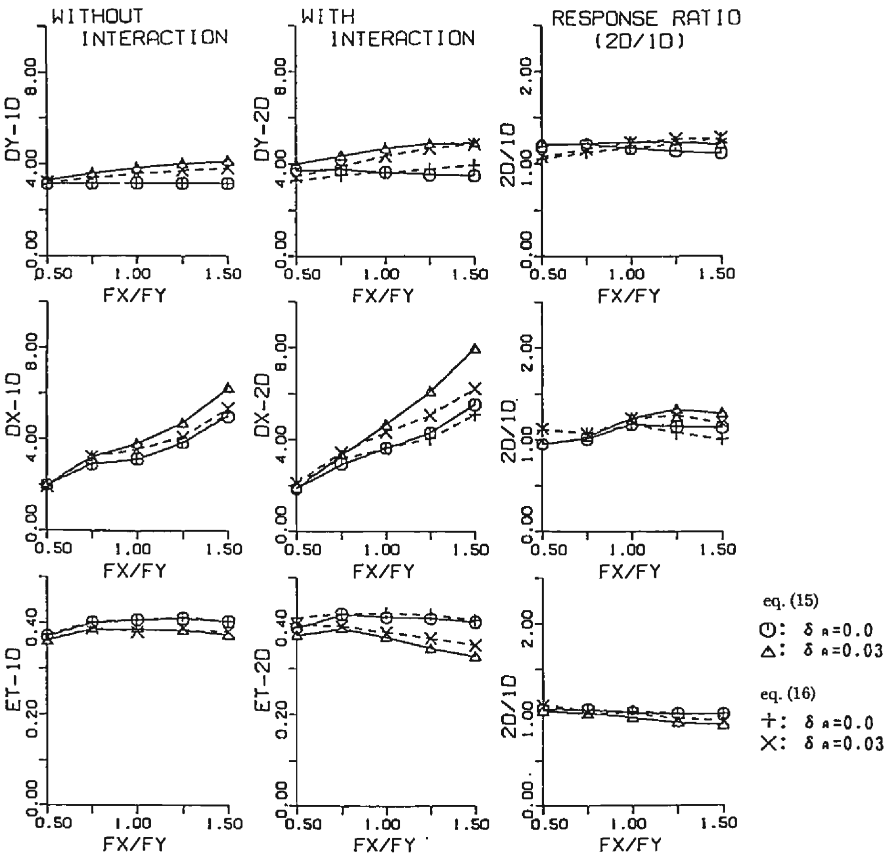
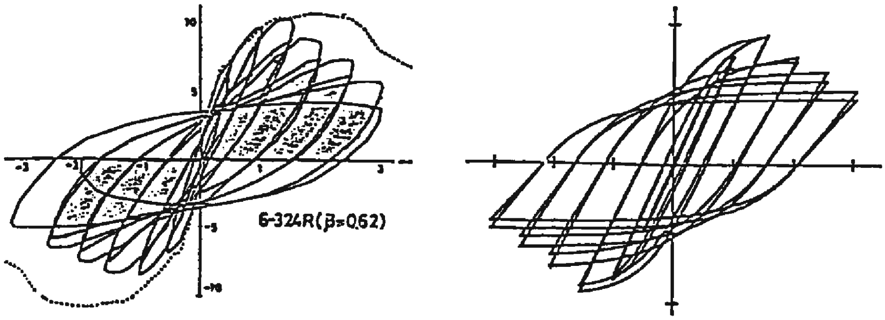


Fig. 4 Comparison of maximum average displacements obtained from deterministic analysis.



(a) experiment (b) model

Fig. 5 Comparison of experimental hysteresis with analysis.

In the subsequent sections, eq. (16) and the above mentioned assumption are used for obtaining a general tendency of the interaction effects under bi-directional ground motion for the sake of simplicity. **Figure 5** shows an example of the restoring force characteristics of a diagonally reinforced concrete column[13] and the model specified by the above formulation in the case of $n=2$.

2.3. Bi-directional horizontal ground motion

Horizontal ground motion considered here, is nonstationary white noise as shown in **Fig. 6**[14].

$$\begin{aligned} f_x(t) &= I(t)\,\xi_x(t) \\ f_y(t) &= I(t)\,\xi_y(t) \end{aligned} \tag{19}$$

where,

$$I(t/t_0)=\bar{a}\,(t/t_0)^{\bar{b}}\exp(-\bar{c}t/t_0),\quad \max I(t/t_0)=1$$
$$\bar{a}=0.18,\ \bar{b}=2.85,\ \bar{c}=0.75,\ t_0=\omega_x T_x=2\pi$$

The spectral characteristics of the soil foundation are represented by a filtered shot-noise

$$S(\omega)=|H_g(\omega)|^2 S_0=\frac{1+4h_g^2(\omega/\omega_g)^2}{\{1-(\omega/\omega_g)^2\}^2+4h_g^2(\omega/\omega_g)^2}S_0 \tag{20}$$

where, S_0 means power spectral density of white noise, and ω_g, h_g show the predominant frequency and damping ratio of the filter, respectively, which are supposed to be the

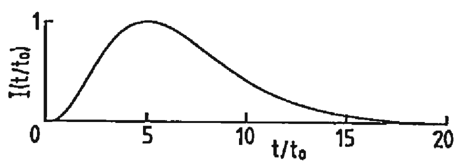


Fig. 6 Envelope function of input motion.

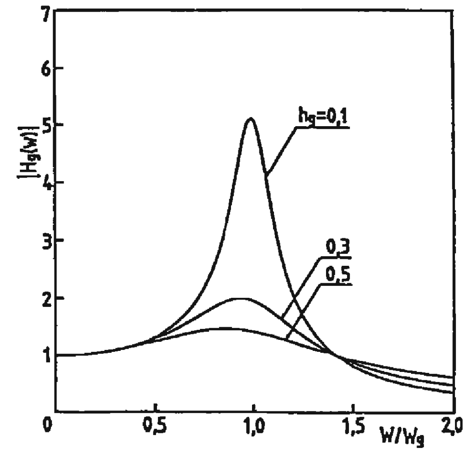


Fig. 7 Spectral characteristics of soil foundation.

same for two directional components without cross correlation. Figure 7 shows a typical spectrum of Kanai-Tajimi filter.

2.4. Equivalent linearization and state equation

Equations of motion of a single-story space structure subjected to bi-directional filtered shot noises are obtained from eqs. (3), (19) and (20).

$$\{\ddot{\mu}\}+2h[\omega]\{\dot{\mu}\}+\alpha[\omega]^2\{\mu\}+(1-\alpha)[\omega]^2\{z\}$$
$$-(\omega_g/\omega_x)^2[\varDelta]\{v\}-2h_g(\omega_g/\omega_x)[\varDelta]\{\dot{v}\}=\{0\} \tag{21}$$
$$\{\ddot{v}\}+2h_g(\omega_g/\omega_x)\{\dot{v}\}+(\omega_g/w_x)^2\{v\}=-I(t)\{\xi(t)\} \tag{22}$$

where

$$\{v\}=\{v_x, v_y\}^T$$
$$\{\xi(t)\}=\{\xi_x(t), \xi_y(t)\}^T$$

Vector z in eq. (21) is expressed as follows by using eq. (16) and the equivalent linearization technique presented by the reference[10,12].

$$\left.\begin{aligned}\dot{z}_x+c_{x1}\dot{\mu}_x+c_{x2}z_x+c_{x3}\dot{\mu}_y+c_{x4}z_y=0\\ \dot{z}_y+c_{y1}\dot{\mu}_y+c_{y2}z_y+c_{y3}\dot{\mu}_x+c_{y4}z_x=0\end{aligned}\right. \tag{23}$$

Those coefficients c_{ij} are given in the Appendix.

The state equation of the system is given from eqs. (21)–(23) by[10]

$$\frac{d}{dt}\{Y\}=[G]\{Y\}+\{F\} \tag{24}$$

where

$$\{Y\}=\{\mu_x, \dot{\mu}_x, z_x, \mu_y, \dot{\mu}_y, z_y, v_x, \dot{v}_x, v_y, \dot{v}_y\}^T$$
$$\{F\}=\{0, 0, 0, 0, 0, 0, 0, -I(t)\xi_x(t), 0, -I(t)\xi_y(t)\}^T$$

$$
[G]=\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\alpha & -2h & -(1-\alpha) & 0 & 0 & 0 & (\omega_g/\omega_x)^2 & 2h_g(\omega_g/\omega_x) & 0 & 0 \\
0 & -c_{x1} & -c_{x2} & 0 & -c_{x3} & -c_{x4} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\alpha(\omega_y/\omega_x)^2 & -2h(\omega_y/\omega_x) & -(1-\alpha)(\omega_y/\omega_x)^2 & 0 & 0 & \Delta_x/\Delta_y(\omega_g/\omega_x)^2 & 2h_g\Delta_x/\Delta_y(\omega_g/\omega_x) \\
0 & -c_{y3} & -c_{y4} & 0 & -c_{y1} & -c_{y2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -(\omega_g/\omega_x)^2 & -2h_g(\omega_g/\omega_x) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(\omega_g/\omega_x)^2 & -2h_g(\omega_g/\omega_x)
\end{pmatrix}
$$

Supposing that input motion be a Gaussian random process, where the mean of each state variable is zero, and the covariance matrix [S] is given by

$$E[\{Y\}]=0, \qquad E[\{F\}]=0 \tag{28}$$

$$[S]=E[(\{Y\}-E[\{Y\}])(\{Y\}-E[\{Y\}])^T]=E[\{Y\}\{Y\}^T] \tag{29}$$

The sum of eq. (24) multiplied by $\{Y\}^T$ from the right and a similar equation of transposition lead to eq. (30)[11],

$$\frac{d}{dt}[S]=[G][S]+[S][G]^T+E[\{F\}\{Y\}^T]+E[\{Y\}\{F\}^T] \tag{30}$$

where

$$
[S]=\begin{pmatrix}
E[\mu_x\mu_x] & E[\mu_x\dot{\mu}_x] & E[\mu_x z_x] & \cdots\cdots & E[\mu_x\dot{v}_y] \\
 & E[\dot{\mu}_x\dot{\mu}_x] & E[\dot{\mu}_x z_x] & \cdots\cdots & E[\dot{\mu}_x\dot{v}_y] \\
 & & E[z_x z_x] & \cdots\cdots & E[z_x\dot{v}_y] \\
 & & & & \cdots\cdots\cdots\cdots\cdots \\
\text{sym.} & & & & E[\dot{v}_y\dot{v}_y]
\end{pmatrix} \tag{31}
$$

and

$$E[\{F\}\{Y\}^T]=E[\{Y\}\{F\}^T]=\frac{1}{2}[B] \tag{32}$$

where

$$B_{88}=I(t)^2 2\pi S_x,\quad B_{1010}=I(t)^2 2\pi S_y,\quad B_{ij}=0 \text{ otherwise}$$

S_x, S_y: power spectral densities of the X and Y directions

Moreover, expectations of eq. (18) are given by

$$\begin{aligned} E[A(e_t)] &= A_0-\delta_A E([e_t] \\ E[\beta(e_t)] &= \beta_0-\delta_\beta E[e_t] \\ E[\gamma(e_t)] &= \gamma_0-\delta_\gamma E[e_t] \end{aligned} \tag{33}$$

$$E[e_t]=\frac{1-\alpha}{1+\Theta}\int_0^t \{E[z_x\dot{\mu}_x]+\Theta E[z_y\dot{\mu}_y]\}\,dt, \tag{34}$$

$$\Theta=(\omega_y/\omega_x)^2(\bar{\Delta}_y/\bar{\Delta}_x)^2$$

3. Stochastic Responses of a Space Structure

In order to investigate the effect of bi-directional ground motion on the response behavior of space structures, the following responses are estimated:

1) Response due to stationary or nonstationary input motions
2) Effect of filter characteristics of soil foundation
3) Response under bi-directional or uni-directional ground motion
4) Response of an isotropic or orthotropic system
5) Responses such as ductility ratio, restoring force, hysteretic energy dissipation, etc.
6) Effect of deterioration of hysteresis

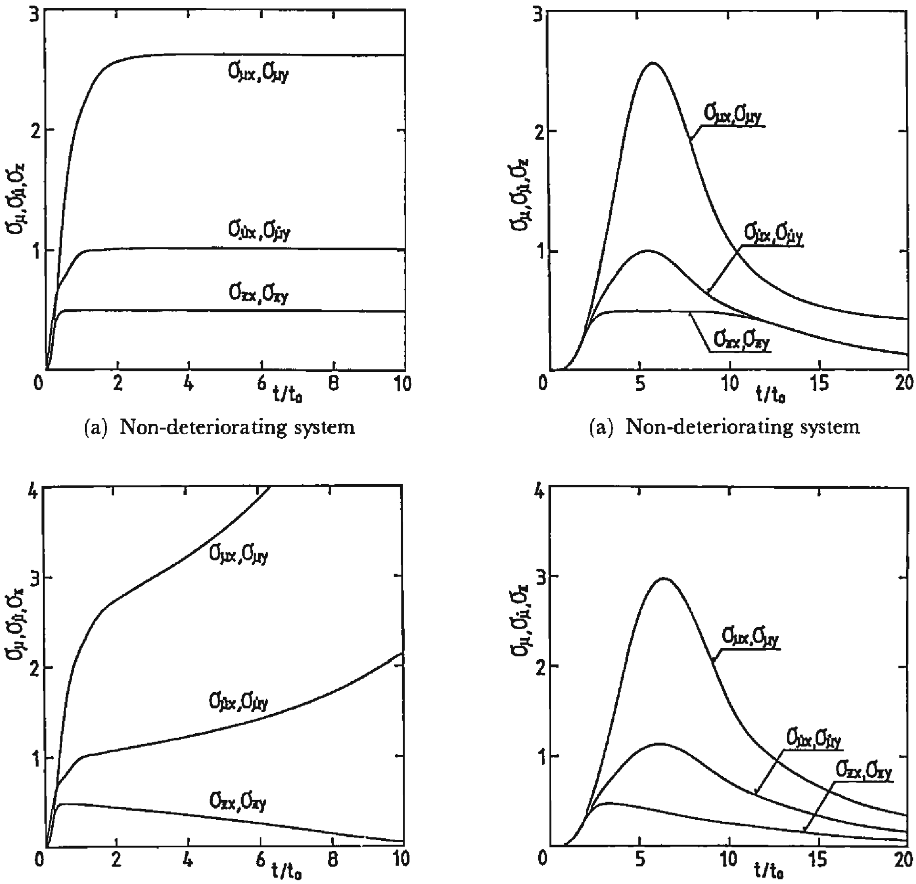
3.1. Responses of isotropic systems

To make clear the fundamental effect of bi-directional motion, time history responses of an isotropic system, that is, $\omega_y/\omega_x=1$, $\bar{\Delta}_y/\bar{\Delta}_x=1$ are shown, which is composed of identical frames in both perpendicular axes under uni-directional motion in a 45° oblique direction to the principal axes.

In **Figs. 8** and **9**, time history responses of the standard deviations of ductility, velocity and restoring force versus nondimensional time are shown in the case of $h=0.02$ $h_g=0.5$ and $\omega_y/\omega_x=1.0$. **Figure 8(b)** shows the effect of deterioration ($\delta_A=0.06$) on the responses under stationary or nonstationary uni-directional motion. ($2\pi S_0=0.6$). The ductility response under stationary input increases rapidly due to deterioration, while restoring force decreases.

The ductility response of a deteriorating system under nonstationary input is about 20% larger than that of a nondeteriorating system.

Figure 9 shows the effects of bi-directional excitations on the ductility response under various levels of ground motion. The ductility response σ_μ of a deteriorating system under bi-directional motion increases considerably as compared with the response $\sigma_{\mu I}$ under uni-directional motion in the case of large value of S_0. On the other hand, the response under uni-directional motion is greater than the response under bi-directional motion in the case of small values of S_0, because of the plastic flow due to interaction effect. This tendency resembles the comparison of elastic



(a) Non-deteriorating system

(b) Deteriorating system ($\delta_A=0.06$)

stationary excitation

(a) Non-deteriorating system

(b) Deteriorating system ($\delta_A=0.06$)

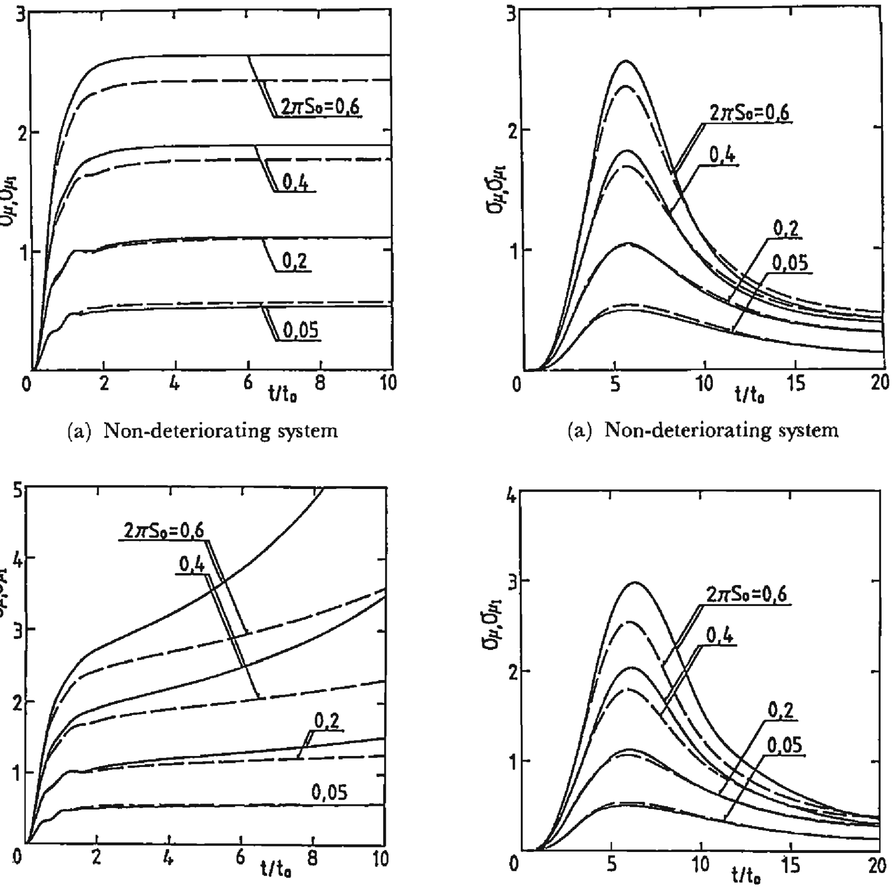
non-stationary excitation

Fig. 8 Standard deviations $\sigma_\mu(t)$, $\sigma_{\dot\mu}(t)$ and $\sigma_z(t)$, $2\pi S_0=0.6$, $h=0.02$, $h_g=0.5$, $\alpha=0.1$.

response with inelastic response.

3.2. Response of orthotropic systems

Two types of orthotropic systems are considered here. In the first type, the frequency ratio ω_y/ω_x varies, while maintaining unit strength ratio $\bar{Q}_y/\bar{Q}_x=1.0$. The second type is a strength ratio varying system with unit frequency ratio $\omega_y/\omega_x=1.0$. All the other parameters are fixed, that is, $h=0.02$, $\alpha=0.1$, $h_g=0.5$, $2\pi S_0=0.6$ and $\delta_A=0.03$. Nondimensional displacements are defined as displacements U_x, U_y in the X- and Y-axis, divided by the limit displacement in the X-axis and nondimensional hysteretic energy dissipation is also defined as hysteretic energy dissipations E_x, E_y in the X- and Y-axis divided by twice the elastic potential energy $\bar{Q}_x\bar{\Delta}_x$ in the X-axis.



(a) Non-deteriorating system (a) Non-deteriorating system

(b) Deteriorating system ($\delta_A=0.06$) (b) Deteriorating system ($\delta_A=0.06$)

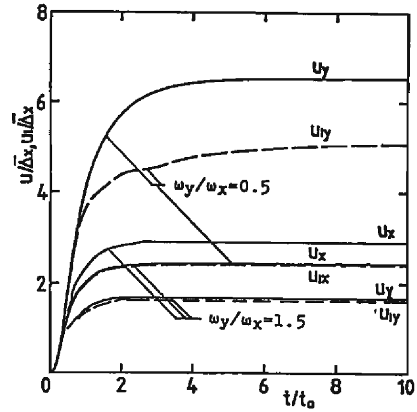
stationary excitation non-stationary excitation

Fig. 9 Standard deviations $\sigma_\mu(t)$, $\sigma_{\mu I}(t)$ with various values of S_0, $h=0.02$, $h_g=0.5$, $\alpha=0.1$.
—— bi-directional inputs, - - - uni-directional input.

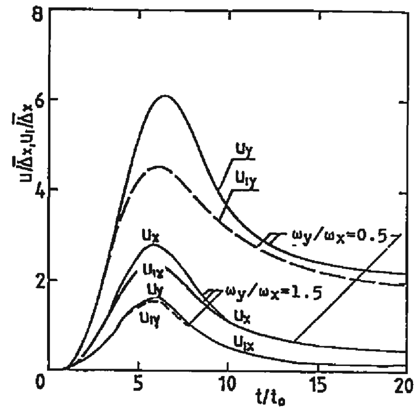
3.2.1. Response of unit strength ratio

In order to make clear the effect of frequency ratio on the responses under bi-directional ground motion, the strength ratio is taken as constant ($\bar{Q}_y/\bar{Q}_x=1.0$) in this section.

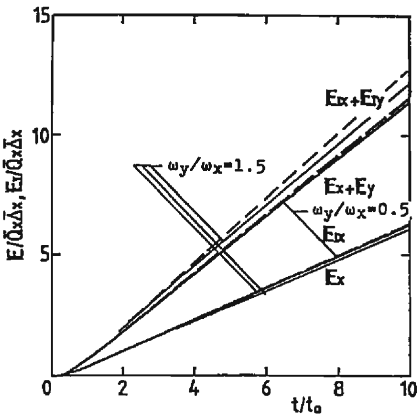
Figure 10 shows standard deviation of displacement and the mean of hysteretic energy dissipation of nondeteriorating space structure with $\omega_y/\omega_x=0.5$ and 1.5 under stationary or nonstationary input motion. In this figure, solid lines show the responses considering interaction effect and broken lines without considering interaction effect. The ratio of natural frequency in the x-axis of the structure to the predominant frequency of input characteristics is supposed to be $\omega_x/\omega_g=1.0$.



(a) Displacement

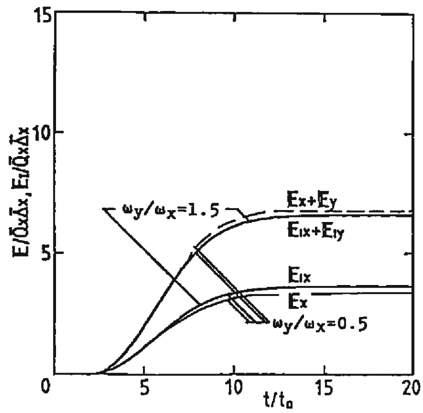


(a) Displacement



(b) Hysteretic energy dissipation

stationary excitation



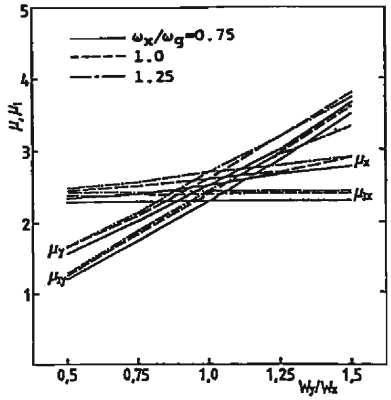
(b) Hysteretic energy dissipation

non-stationary excitation

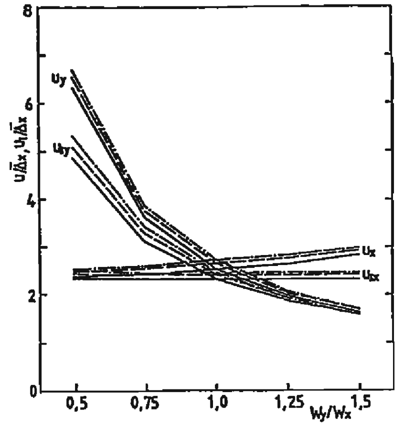
Fig. 10 Standard deviation of displacement and mean of hysteretic energy dissipation of a non-deteriorating system, $2\pi S_0=0.6$.
—— bi-directional input, - - - uni-directional input.

Displacement response under bi-directional excitation increases especially for the smaller frequency ratio, while hysteretic energy dissipation under bi-directional excitation is similar to or smaller than the response under uni-directional motion. Similar results are obtained in the case of nonstationary input motion. In this case, the peak value of displacement in the smaller frequency axis is 1.6–1.2 times larger due to interaction effect.

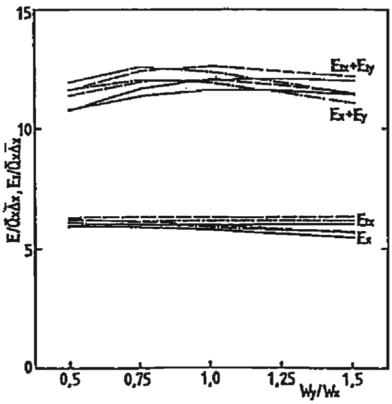
Figure 11 shows the variation in the interactive effect according to the frequency characteristics of space structure and input motion. From this figure, it is found that the interactive effect is larger in the direction of the smaller frequency, in which



(a) Ductility



(c) Displacement

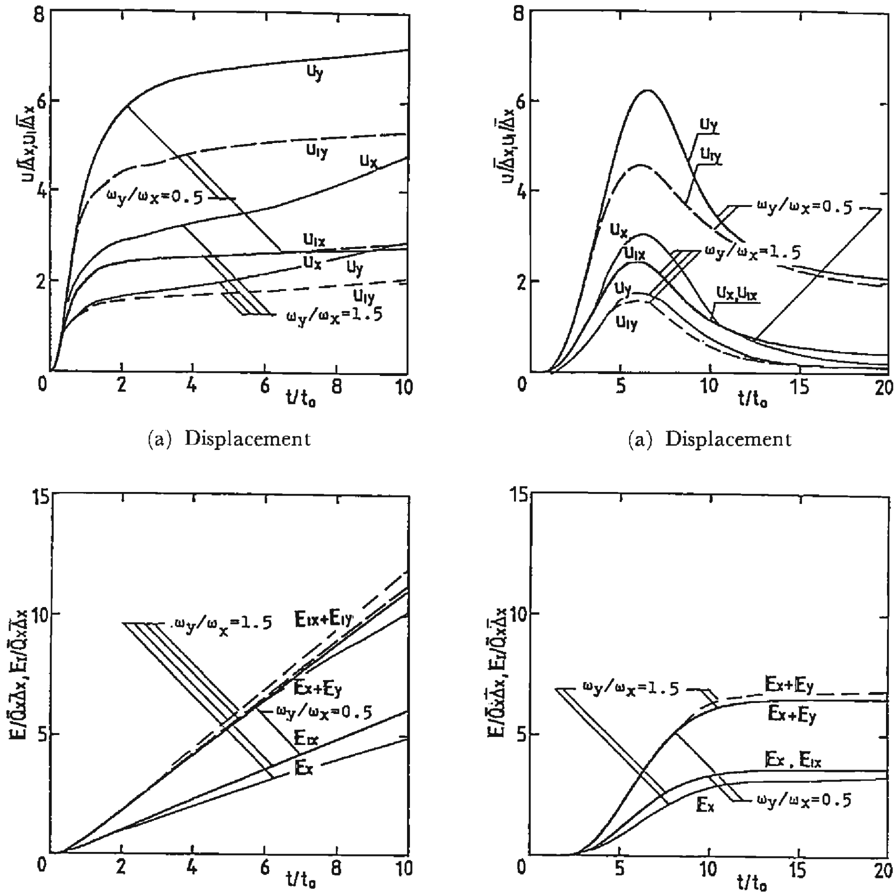


(b) Hysteretic energy dissipation

Fig. 11 Standard deviation of ductility and displacement and mean of hysteretic energy dissipation of a non-deteriorating system under stationary excitations with various values of ω_y/ω_x, $2\pi S_0=0.6$.
—— bi-directional input, - - - uni-directional input.

a smaller ductility response is as expected under uni-directional motion. The response of hysteretic energy dissipation is rather uniform for various values of ω_y/ω_x and ω_x/ω_g in spite of a large difference in displacement response, the reason for which may be interpreted as a large deformation is induced by a decrease in restoring force due to interaction.

Figure 12 and 13 show the responses of deteriorating system ($\delta_A=0.03$) which correspond to nondeteriorating responses shown in **Figs. 10** and **11**, respectively. The displacement response of a deteriorating system under stationary input gradually increases with time but is similar to a nondeteriorating response when $\omega_y/\omega_x=0.5$, that is, the natural frequency of the Y-axis is less than the predominant frequency of input motion. On the other hand, when $\omega_y/\omega_x=1.5$, that is, the natural frequency of the Y-axis is higher than the predominant frequency of input motion, displacement responses not only in the Y-axis but X-axis remarkably increase due to bi-directional



(a) Displacement (a) Displacement

(b) Hysteretic energy dissipation (b) Hysteretic energy dissipation

stationary excitation non-stationary excitation

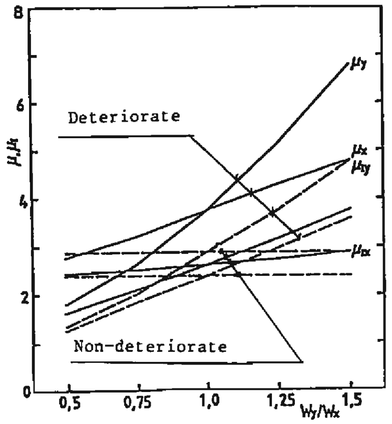
Fig. 12 Standard deviation of displacement and mean of hysteretic energy dissipation of a deteriorating system, (δ_A=0.03), $2\pi S_0$=0.6.
——bi-directional input, - - - uni-directional input.

ground motion. This tendency may become more significant when predominant frequency of the ground motion is smaller than the frequency of the space structure with weaker stiffness. The response of hysteretic energy dissipation of deteriorating systems is nearly equal to or less than that of nondeteriorating systems.

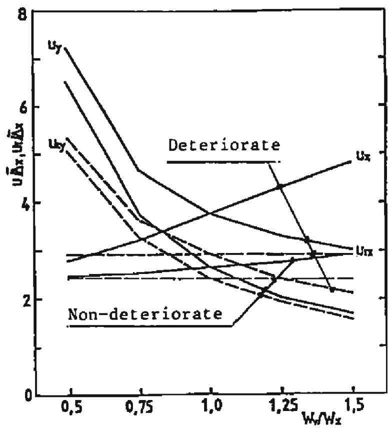
As shown in **Fig. 13**, the ductility response increases more than 50% due to the interactive effect in both directions in the case of deteriorating systems, as compared with the case of nondeteriorating systems.

3.2.2. Response of unit frequency ratio

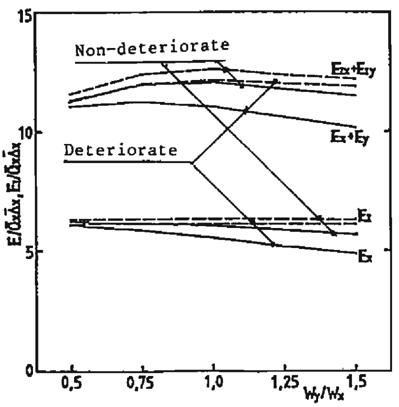
In this section, the frequency ratio is assumed to be unity ($\omega_y/\omega_x=1$) for clarifying



(a) Ductility



(c) Displacement

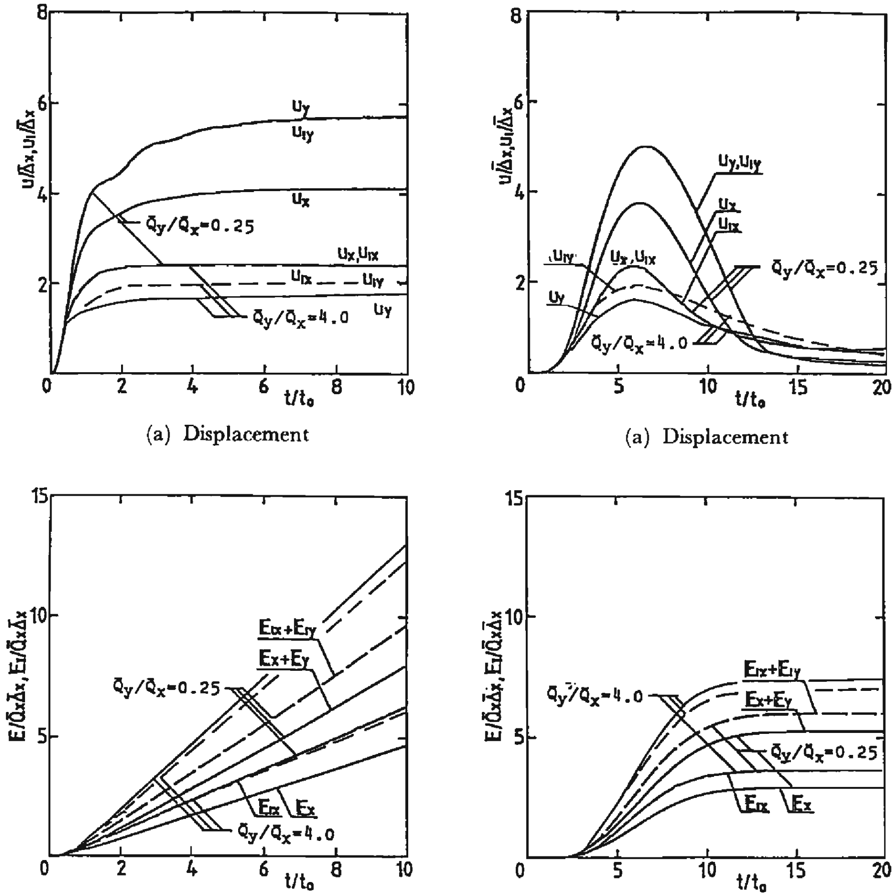


(b) Hysteretic energy dissipation

Fig. 13 Standard deviation of ductility and displacement and mean of hysteretic energy dissipation of a deteriorating system under stationary excitation with various values of ω_y/ω_x, $2\pi S_0=0.6$, $\omega_x/\omega_g=1.0$.
—— bi-directional input, --- uni-directional input

the effect of strength ratio $\bar{Q}_y/\bar{Q}_x$. Other parameters have the same values as those in the previous section except for $\bar{Q}_y/\bar{Q}_x$. **Fih. 14** shows the time history of displacement and hysteretic energy dissipation in the cases of $\bar{Q}_y/\bar{Q}_x=0.25$ and 4.0. Displacement in the X-axis under bi-directional excitation increases to about twice that under uni-directional motion in the case of $\bar{Q}_y/\bar{Q}_x=0.25$, while the response considering interaction becomes smaller than that under uni-directional motion in the case of $\bar{Q}_y/\bar{Q}_x=4.0$.

Figure 15 summarizes the effect of strength ratio between perpendicular axes on the response of nondeteriorating systems under stationary input. The effect of interaction on the ductility response is large in the stronger axis along which smaller ductility response may develop under uni-axial motion. As shown in **Fig. 15(a)** the ductility response in the Y-axis is less than 1 in the case of $\bar{Q}_y/\bar{Q}_x=4$. That is the reason why the displacement in the stronger axis under bi-directional input is less than



(a) Displacement

(a) Displacement

(b) Hysteretic energy dissipation

(b) Hysteretic energy dissipation

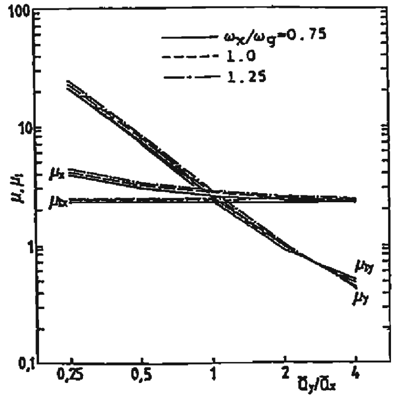
stationary excitation

non-stationary excitation

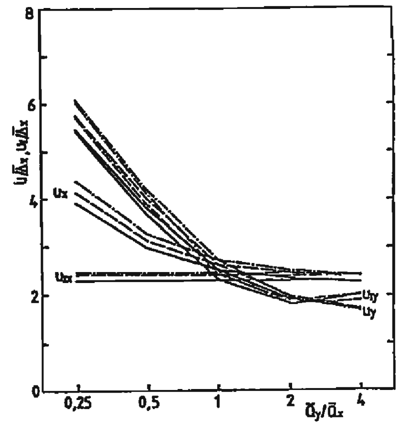
Fig. 14 Standard deviation of displacement and mean of hysteretic energy dissipation of non-deteriorating system, $2\pi S_0=0.6$.
—— bi-directional input, --- uni-directional input.

the response under uni-directional input.

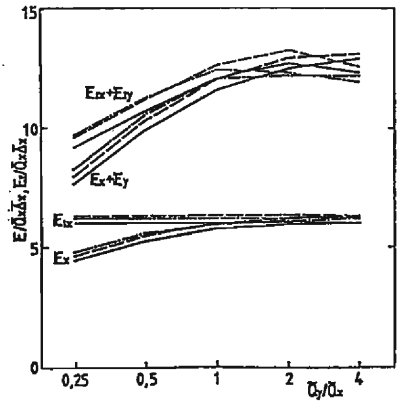
The last two figures (**Figs. 16 and 17**) show the responses considering deterioration effect compared with the responses without deterioration shown in **Figs. 14** and **15.** The interaction effect appears remarkably in both axes and displacement and ductility responses considering deterioration increases not only in the stronger axis but also in the weaker axis. On the other hand, the response of hysteretic energy dissipation considering deterioration decreases considerably because of the deterioration of restoring forces.



(a) Ductility



(c) Displacement



(b) Hysteretic energy dissipation

Fig. 15 Standard deviation of ductility and displacement and mean of hysteretic energy dissipation of a non-deteriorating system under stationary excitation with various values of Q_y/Q_x, $2\pi S_0=0.6$.
—— bi-directional input, --- uni-directional input

4. Concluding remarks

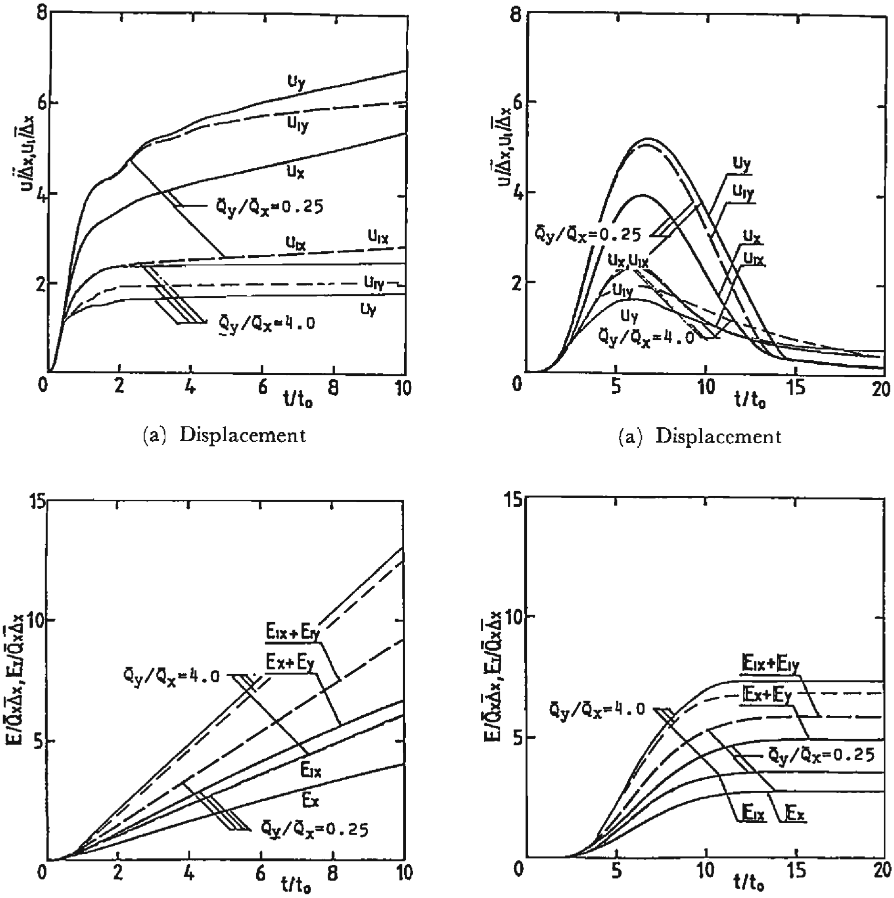
Random responses of space structures subjected to bi-directional horizontal ground motion were analyzed in order to make clear the ultimate aseismic safety and compared with responses under uni-directional ground motion. The formulation of potential function was presented by considering the general interaction rule in plasticity.

In order to make clear the effect of bi-directional ground motion on the inelastic responses of space structures, a parametric study was performed.

From the restricted results, the following observations were made:

1) Maximum displacement response calculated from eq. (15) is rather similar to that calculated from the equation presented by Y. K. Wen et al.

2) The formulation of deteriorating hystereses presented by Baber and Wen duplicates the experimental result of a diagonally reinforced concrete column.



(a) Displacement (a) Displacement

(b) Hysteretic energy dissipation (b) Hysteretic energy dissipation

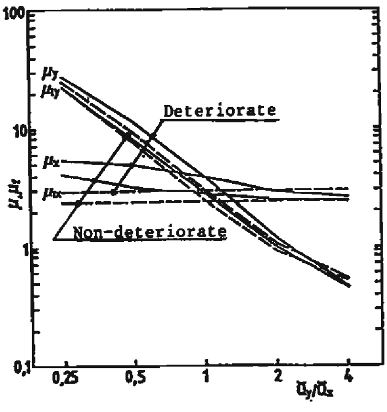
stationary excitation non-stationary excitation

Fig. 16 Standard deviation of displacement and mean of hysteretic energy dissipation of a deteriorating system, ($\phi_A=0.03$), $2\pi S_0=0.6$.
——bi-directional input, - - - uni-directional input.

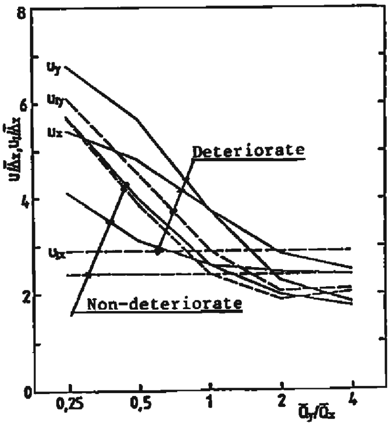
3) In the case of great excitation, the responses of structures having higher natural frequency than the predominant frequency of input motions become large because of elongation of the inelastic period of structures.

4) In the case of relatively lesser excitation, responses under bi-directional motion become smaller than those under uni-directional motion because of hysteretic energy dissipation due to bi-axial interaction.

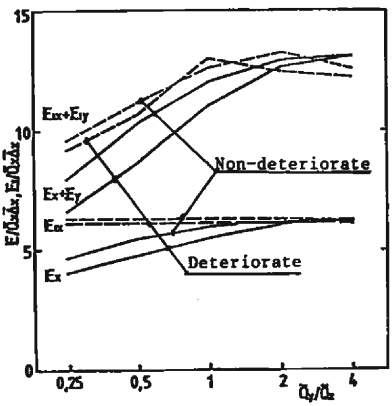
5) The ductility responses of isotropic deteriorating systems under stationary input considerably increase and about 20% increase under nonstationary input as compared with the case of nondeteriorating systems.



(a) Ductility



(c) Displacement



(b) Hysteretic energy dissipation

Fig. 17 Standard deviation of ductility and displacement and mean of hysteretic energy dissipation of a deteriotating system under stationary excitation with various values of Q_y/Q_x, $2\pi S_0=0.6$, $\omega_x/\omega_y=1.0$. —— bi-directional input, - - - uni-directional input

6) In the case of orthotropic systems with unit strength ratios, the interaction effect is mainly on the displacement response in the direction of the lower frequency axis of a structure along which a smaller ductility response is expected under uni-directional motion. On the other hand, the response of hysteretic energy dissipation under bi-directional motion decreases due to plastic deformation at relatively low level of interactive restoring forces.

7) The effect of deterioration is remarkable, for instance, the displacement response becomes about twice the response under uni-directional motion in the case of unit frequency ratio.

From the above results, considering that the column members at the base inevitably behave inelastically under severe earthquake ground motion, it is important to note that such members must possess enough deformation capacity, especially, reinforced concrete columns which often deteriorate.

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Appendix

Linearlization of the hysteretic formulation is as follows:

$$\begin{aligned} &\dot{z}_x+g_x(\dot{\mu}, z)=0 \\ &\dot{z}_y+g_y(\dot{\mu}, z)=0 \end{aligned} \tag{A-1}$$

When β is equal to γ, eq. (16) becomes

$$\begin{aligned} g_x(\dot{\mu}, z)=&\gamma(z_x^2+z_y^2)^{(n-2)/2} \\ &\{|\dot{\mu}_x z_x|z_x+\dot{\mu}_x z_x^2+|\dot{\mu}_y z_y|z_x+\dot{\mu}_y z_x z_y\}-A\dot{\mu}_x \\ g_y(\dot{\mu}, z)=&\gamma(\tau_x^2+\tau_y^2)^{(n-2)/2} \\ &\{|\dot{\mu}_y z_y|z_y+\dot{\mu}_y z_y^2+|\dot{\mu}_x z_x|z_y+\dot{\mu}_x z_y z_x\}-A\dot{\mu}_y \end{aligned} \tag{A-2}$$

Replacing $m=(n-2)/2$ (m; integer), and considering

$$(z_x^2+z_y^2)^m=\sum_{r=0}^{m}\binom{m}{r}z_x^{2(m-r)}z_y^{2r} \tag{A-3}$$

eq. (A-2) can be written as

$$\begin{aligned} g_x(\dot{\mu}, z)=&\gamma\sum_{r=0}^{m}\binom{m}{r}\{|\dot{\mu}_x z_x|z_x^{2(m-r)+1}z_y^{2r}+\dot{\mu}_x z_x^{2(m-r+1)}z_y^{2r} \\ &+|\dot{\mu}_y z_y|z_x^{2(m-r)+1}z_y^{2r}+\dot{\mu}_y z_x^{2(m-r)+1}z_y^{2r+1}\}-A\dot{\mu}_x \\ g_y(\dot{\mu}, z)=&\gamma\sum_{r=0}^{m}\binom{m}{r}\{|\dot{\mu}_y z_y|z_y^{2(m-r)+1}z_x^{2r}+\dot{\mu}_y z_y^{2(m-r+1)}z_x^{2r} \\ &+|\dot{\mu}_x z_x|z_y^{2(m-r)+1}z_x^{2r}+\dot{\mu}_y z_y^{2(m-r)+1}z_x^{2r+1}\}-A\dot{\mu}_y \end{aligned} \tag{A-4}$$

Coefficients of eq. (23) are calculated from the following equations.

$$\begin{aligned} c_{x1}=&E\left[\frac{\partial}{\partial\dot{\mu}_x}g_x(\dot{\mu}, z)\right] \\ =&\gamma\sum_{r=0}^{m}\binom{m}{r}\{E[\mathrm{sgn}(\dot{\mu}_x)|z_x|z_x^{2(m-r)+1}z_y^{2r}]+E[z_x^{2(m-r+1)}z_y^{2r}]\}-A \\ c_{x2}=&E\left[\frac{\partial}{\partial z_x}g_x(\dot{\mu}, z)\right] \\ =&\gamma\sum_{r=0}^{m}\binom{m}{r}2(m-r+1)\{E[|\dot{\mu}_x z_x|z_x^{2(m-r)}z_y^{2r}] \\ &+E[\dot{\mu}_x z_x^{2(m-r)+1}z_y^{2r}]\} \\ &+\gamma\sum_{r=0}^{m}\binom{m}{r}\{2(m-r)+1\}\{E[\dot{\mu}_y z_y z_x^{2(m-r)}z_y^{2r}] \\ &+E[\dot{\mu}_y z_x^{2(m-r)}z_y^{2r+1}]\} \end{aligned} \tag{A-5}$$

$$c_{x3}=E\left[\frac{\partial}{\partial\dot{\mu}_y}g_x(\dot{\mu}, z)\right]$$

$$=\gamma\sum_{r=0}^{m}\binom{m}{r}\{E[\mathrm{sgn}(\dot{\mu}_y)|z_y|z_x^{2(m-r)+1}z_y^{2r}]+E[z_x^{2(m-r)+1}z_y^{2r+1}]\}$$

$$c_{x4}=E\left[\frac{\partial}{\partial z_y}g_x(\dot{\mu},z)\right]$$

$$=\gamma\sum_{r=0}^{m}\binom{m}{r}\{2rE[|\dot{\mu}_x z_x|z_x^{2(m-r)+1}z_y^{2r-1}]$$

$$+2rE[\dot{\mu}_x z_x^{2(m-r+1)}z_y^{2r-1}]$$

$$+(2r+1)E[|\dot{\mu}_y z_y|z_x^{2(m-r)+1}z_y^{2r-1}]$$

$$+(2r+1)E[\dot{\mu}_y z_x^{2(m-r)+1}z_y^{2r}]\}$$

Similar relationships with regard to c_{yj} are obtained as eq. (A–5).